b- $\forall x \in \mathbb{R}^+$

$$G(x) = \int_{Log2}^{x} t^2e^{-t}dt \quad \left(\begin{array}{lll} u_1'(t)=e^{-t} & \rightarrowtail & u_1(t)=-e^{-t} \\ v_1(t) = t^2 & \rightarrowtail & v_1'(t) = 2t \end{array} \right)$$

$$= [-t^2e^{-t}]_{\ln 2}^{x} + 2\int_{Log2}^{x} te^{-t}dt$$

$$\left(\begin{array}{lll} u_2'(t)=e^{-t} & \rightarrowtail & u_2(t)=-e^{-t} \\ v_2(t)=t & \rightarrowtail & v_2'(t) = 1 \end{array} \right)$$

$$= -x^2e^{-x} + e^{-\ln 2}(\ln 2)^2 + 2\left[[-te^{-t}]_{\ln 2}^{x} + \int_{Log2}^{x} e^{-t}dt \right]$$

$$= -x^2e^{-x} + e^{-\ln 2}(\ln 2)^2 - 2xe^{-x} + 2\ln 2e^{-\ln 2} + 2[-e^{-t}]_{\ln 2}^{x}$$

$$= -x^2e^{-x} + \frac{1}{2}(\ln 2)^2 - 2xe^{-x} + \ln 2 - 2e^{-x} + 1$$

$$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + \frac{1}{2}(\ln 2)^2 + \ln 2 + 1.$$

$$\cdot \lim_{x\to+\infty} G(x) = \lim_{x\to+\infty} (-x^2e^{-x} - 2xe^{-x} - 2e^{-x}) + \frac{1}{2}(\ln 2)^2 + \ln 2 + 1$$

(on pose $X = -x$; quand $x \to +\infty; X \to -\infty$)

$$= \lim_{X\to-\infty} [-X^2e^{X} + 2Xe^{X} - 2e^{X}] + \frac{1}{2}(\ln 2)^2 + \ln 2 + 1.$$

$$= -\lim_{X\to-\infty} \left(Xe^{\frac{X}{2}} \right)^2 + \frac{1}{2}(\ln 2)^2 + \ln 2 + 1$$

(car $\lim\limits_{X\to-\infty} Xe^{X} = 0$ et $\lim\limits_{X\to-\infty} e^{X} = 0$)

$$= -\lim_{y\to-\infty} [(2ye^{y})^2] + \frac{1}{2}(\ln 2)^2 + \ln 2 + 1 \quad \text{avec } y=\frac{X}{2}$$

$$= \frac{1}{2}(\ln 2)^2 + \ln 2 + 1.$$

c- $\forall t \in [Log2, +\infty[$ on a :

$$f(t) - 2t^2e^{-t} = t^2\left[\frac{1}{e^t - 1} - 2e^{-t} \right] = t^2\left[\frac{1 - 2e^{-t}(e^t - 1)}{e^t - 1} \right]$$

$$= t^2\left[\frac{-1 + 2e^{-t}}{e^t - 1} \right].$$

$\cdot\ t \geq Log2 \Leftrightarrow -t \leq Log\frac{1}{2} \Leftrightarrow e^{-t} \leq \frac{1}{2} \Leftrightarrow 2e^{-t} - 1 \leq 0.$

$\cdot\ t \geq Log2 \Rightarrow t^2 > 0$ et $e^t - 1 > 0$

D'où $f(t) - 2t^2e^{-t} \leq 0 \Leftrightarrow f(t) \leq 2t^2e^{-t}$.

$\otimes$ Soit $x \in [Log2, +\infty[$ on a

$\forall t \in [Log2, +\infty[; f(t) \leq 2t^2e^{-t}$

$$\Rightarrow \int_{Log2}^{x} f(t)dt \leq 2\int_{Log2}^{x} t^2e^{-t}dt$$

$$\text{Or } F(x) = \int_0^x f(t)dt = \int_0^{Log2} f(t)dt + \int_{Log2}^{x} f(t)dt$$

$$\text{D'où } F(x) = \int_0^{Log2} f(t)dt + \int_{Log2}^{x} f(t)dt \leq F(Log2) + 2G(x)$$